AP Calculus AB

WS 9.7 – Exponential Growth

1) $\frac{dN}{dt} = \frac{1}{5}N$, $N(0) = 500$

$N(t) = Ce^{kt}$

$N(t) = 500e^{\frac{1}{5}t}$

2) $\frac{dN}{dt} = \frac{1}{2}N$

$N(t) = 1000e^{\frac{1}{2}t}$

3) $\frac{dM}{dt} = -\frac{M}{50}$

$N(t) = 250e^{-\frac{1}{50}t}$

$M(100\ln 2) = 250e^{-\frac{1}{50}(100\ln 2)}$

$= 250e^{-2\ln 2}$

$= 250e^{\ln 2^{-2}}$

$= 250\left(\frac{1}{4}\right)$

$= 62.5$ g.

4) $f(t) = \int_0^{t^2} \frac{1}{1+x^2}\,dx$

$f'(t) = \frac{1}{1+t^4} \cdot 2t$

$= \frac{2t}{1+t^4}$

5) $f(x) = x^4 - 4x^3 + 6$ on $[1, 4]$

$f'(x) = 4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

~~$x = 0$~~ $x = 3$

x	$f(x)$
1	3
3	−21
4	6

$f(x)$ has a max of $y = 6$ @ $x = 4$.

6) $\frac{dP}{dt} = \frac{1}{10}P$

$P(t) = 500e^{\frac{1}{10}t}$

$P(20) = 500e^2 = 3694.528$

≈ 3695

7) $f'(2.1) \approx \frac{f(2.2) - f(2.0)}{2.2 - 2.0}$

$= \frac{1.73 - 1.39}{0.2}$

$= 1.7$

8) $x(8) = 10$

$x(5) = 10 + \int_8^5 v(t)\,dt$

$= 10 + \left[\frac{1}{2}(2)(4) - \frac{1}{2}(1)(2)\right]$

$= 13$

9) $v(t)=0$ @ $t=6, 9$

$\int_0^6 v(t) = \frac{1}{2}(6+3)(2) = 9$ to the right of $x(0)$.

$\int_0^9 v(t) = 9 - \frac{1}{2}(4)(3) = 3$ to the right of $x(0)$

At $t=6$, the object is farthest from $x(0)$

10) The object is moving left on $(6,9)$ b/c $v(t)<0$

11) $ROC_{avg} = \frac{f(4)-f(1)}{4-1}$

$= \frac{6-2}{3}$

$= \frac{4}{3}$

12) $h(x) = g(f(x))$

$h'(x) = g'(f(x)) \cdot f'(x)$

$h'(3) = g'(f(3)) \cdot f'(3)$

$= g'(4) \cdot f'(3)$

$= \left(\frac{1}{2}\right)(2)$

$= 1$